A close examination of the terminal productions reveals that we can replace these by simpler productions that correspond to only one step:

\[
\begin{align*}
H & \leftarrow H \uparrow J \rightarrow L \rightarrow H & \uparrow \\
J & \leftarrow J \leftarrow K \leftarrow H \uparrow J & \uparrow \\
K & \leftarrow K \downarrow L \leftarrow J \leftarrow K & \downarrow \\
L & \leftarrow L \rightarrow H \rightarrow K \downarrow L & \downarrow 
\end{align*}
\]

One expansion step on the non-terminals immediately brings back the original productions.

\section*{From the H-order to the Sierpinski Curve}

If we remember the grammar-based description of the Sierpinski curve,

\[
\begin{align*}
S & \leftarrow S \uparrow R \rightarrow P \swarrow S \\
R & \leftarrow R \searrow Z \uparrow S \swarrow R \\
Z & \leftarrow Z \searrow P \leftarrow R \searrow Z \\
P & \leftarrow P \swarrow S \downarrow Z \swarrow P 
\end{align*}
\]

we recognise a strong similarity to the grammar for the H-index. The recursion scheme for the non-terminals is equivalent, if we map the symbols \( S \rightarrow H \), \( R \rightarrow J \), \( Z \rightarrow K \), and \( P \rightarrow L \). This structural equivalence is due to the triangle-based construction: in both constructions, the triangles are subdivided according to the same scheme and the geometrical orientation of the triangles determines the pattern of the curve. In addition, the child triangles are traversed in the same order – which is not too surprising, as there is, in fact, no other choice, if we require adjacent triangles to share a common edge.

Figure 7.6 shows a comparison of the H-index with the iterations (left image) and with the approximating polygons (right image) of the Sierpinski curve. We can obviously obtain the H-index traversal by replacing all diagonal steps of the Sierpinski iteration by two steps that are in horizontal and vertical direction. And we also see that all nodes of the H-index iteration are also nodes of the approximating polygon of the Sierpinski curve. Moreover these H-index nodes are visited in Sierpinski order. We can therefore interpret the H-index iterations as polygons that approximate the Sierpinski curve, which implies that the H-index will not lead to a new space-filling curve. Its infinite iteration will lead to the Sierpinski curve, again.